LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 03

B.Sc. DEGREE EXAMINATION **– MATHEMATICS**

THIRD SEMESTER **– NOVEMBER 2013**

MT 3503 - VECTOR ANALYSIS AND ORDINARY DIFFERENTIAL EQUATION.

Date: 06/11/2013 **Dept. No.:** ____ **Max. 100 Marks**

Time: 9.00 – 12.00

SECTION – A

(Answer ALL questions) **(10 × 2 = 20)**

1. If $\phi(x,y,z) = x^2y + y^2x + z^2$, find $\nabla\varphi$ at (1,1,1).
2. If $\bar{F} = xy^2\vec{i} + 2x^2yz\vec{j} - 3yz^2\vec{k}$, find div $\bar{F}$.
3. Define line integral of a conservative vector.
4. What are spherical coordinates?
5. State Gauss divergent theorem.
6. State Stroke's theorem.
7. Solve $\frac{dy}{dx} = \frac{y+2}{x-1}$.
8. Find the general solution of $y = xp + \frac{\alpha}{p}$.
9. Solve $(D^2 + 6D + 9)y=0$.
10. Find the complimentary function of $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$.

SECTION -B **(5X8=40)**

(Answer any FIVE questions)

11. Find the directional derivative of $\phi = 3x^2 + y - 3z$ at (1,1,1) in the direction of $2\vec{i} + 2\vec{j} - \vec{k}$.
12. If $\vec{F} = 3xyz^2\vec{i} + 2xy^3\vec{j} - x^2yz\vec{k}$ and $f = 3x^2 - yz$ find (i) $\vec{F} \cdot \nabla f$ (ii) $\nabla \cdot (\nabla f)$ at (1,-1,1).
13. If $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$, then evaluate $\iiint_V \vec{F}\, dv$ where V is the region bounded by the surfaces x = 0 , y = 0 , y = 6 , z = x^2 ,z = 4 .
14. Using Divergence theorem evaluate $\iint_S \vec{F} \cdot \vec{n}\, ds$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounded by the region $x^2 + y^2 = 4$, z = 0 and z = 3.
15. Solve $x^2p^2 + 3xyp + 2y^2 = 0$.
16. Solve $xp^2 - yp - x = 0$.
17. Solve $(D^2 - 2D + 2)y = e^x x^2 + 5 + e^{-2x}$.
18. Solve$(x^2 D^2 - 3xD - 5)y = \cos(\log x)$

SECTION – C

(Answer any TWO questions) **(2 × 20 = 40)**

19. (a) If □φ = $(y^2 - 2xyz^3)\vec{i} + (3+2xy- x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$, find φ .

(b)Prove that $\square \times (\square \times \vec{F}) = \nabla(\square.\vec{F}) - \nabla^2 \vec{F}$.

(c) Find a unit normal vector to the surface $x^2 + xy + z^2 = 4$ at the point (1,-1,2). (7+7+6)

20. Verify Stroke's theorem for the vector field defined by $\vec{F} = (x^2 - y^2)\,\vec{i} + xy\,\vec{j}$ in the region in XY plane bounded by x=0; x=a; y=0; and y=b.

21 (a) Solve $\frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy}$.

(b) Solve $(1 + x^2)\frac{dy}{dx} + y = 1$. (7+6+7)

(c) Solve $yp^2 - xp + 2y = 0$.

22 (a) Solve $\frac{d^2y}{dx^2} + y = \tan x$.

(b) Solve $(x^2D^2 - 3xD)y = x+1$. (10+10)

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